Speed Round
NC(SMC)^2
2022

Speed Round

Instructions: This round contains 30 problems, and a time limit of 40 minutes. Submit your answers here: https://forms.gle/qTTwHPySfSSHUHkaA. **All answers are integers.** Good Luck!

1. What is the value of the expression $2 - \frac{2}{2-\frac{2}{2}}$?

2. Willow counts downwards by 7's, starting from $100, 93, 86, \ldots$. What is the 7th number she says?

3. Gabriela has a basket containing apples and pears, and she has twice as many pears as apples. If the basket contains 9 fruits in total, how many pears are there?

4. 30% of 50 is equal to what percent of 30?

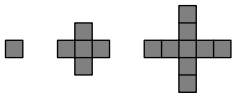
5. Jeffrey spent \$20 on oranges, clementines, and grapefruits. Each of the fruits costs \$2 apiece. If Jeffrey bought 3 oranges and 4 clementines, how many grapefruits did he buy?

6. How many two digit numbers are divisible by 3?

7. Isaac and Eli are each trying to approximate the value of the sum $17622 + 43721$.
   - Isaac rounds both of the numbers to the nearest thousand, and then adds them together.
   - Eli adds the numbers together, and then rounds the result to the nearest thousand.

   What is the positive difference between Isaac's result and Eli's result?

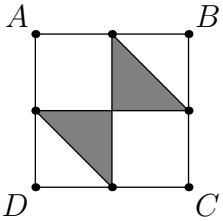
8. Consider a sequence of shapes, the first three of which are as follows:

   How many squares are in the tenth shape?

9. At NCSSM, 3 cups of boba are worth 2 bags of ramen, and 1 bag of ramen is worth 9 ping pong balls. How many ping pong balls are 9 cups of boba worth?

10. Jonathan draws a square $ABCD$ of side length 4, and then marks the midpoints of the four sides as shown. What is the total area of the shaded region?

11. The numbers $3, 4, 5, 6$ each go in one of the four blanks below:

$$\_\_ \times \_\_ - \_\_ \times \_\_$$

What is the largest possible value of the above expression, over all ways to fill in the blanks?

12. If 2022 workers can dig 2022 holes in 2022 days, how many days does it take 2023 workers to dig 2023 holes?

13. Philip took four tests, and his average score was 90. If he scores 100 on his next test, what will his average score be across the five tests?

14. Angie and Bonnie are 100 meters apart from each other, and running towards each other. Angie runs at a speed of 4 meters per second, and Bonnie runs at a speed of 6 meters per second. How many seconds will it take for them to run into each other?

15. Isaac buys a ruler and a compass for a total price of \$2.20. If the compass cost \$1.00 more than the ruler, how many cents did the ruler cost?

16. Kevin owns 12 buffalo, 3 of which are albino. Nathan owns 24 buffalo, some of which are also albino. Kevin and Nathan notice that when all their buffalo are combined, 50% of them are albino. How many of Nathan's buffalo are albino?

17. Two old cat ladies live in a house together. The first cat lady has 10 adult cats, each of which has 10 kittens, each of which has 10 toys. The second cat lady has 9 adult cats, each of which has 9 kittens, each of which has 9 toys.

    One day, all of the kittens misbehaved, so the adult cats confiscated all of the toys and distributed them among themselves. On average, how many toys went to each adult cat?

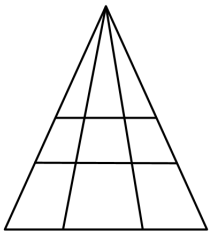
18. Out of the 500 students at NCSSM, 258 are Juniors and 242 are Seniors. 342 students like math, while the other 158 of them don't. If 181 of the Juniors like math, how many of the Seniors don't like math?

19. A quadrilateral $ABCD$ has $AB = BC, CD = DA$, and $AC = BD$. How many of the following statements must be true?

    - The diagonals of the quadrilateral are perpendicular.
    - All four sides of the quadrilateral are equal.
    - The quadrilateral is a parallelogram.
    - The quadrilateral is a square.

20. In a basketball game, Eli attempted four times as many 2-pointer shots as 3-pointer shots. In addition, his accuracy on 3-pointer shots was half his accuracy on 2-pointer shots. If Eli scored 32 points from making 2-pointer shots, how many total points did he score in the game?

21. How many triangles are there in the following image?



22. I have enough money to buy 12 apples. This amount of money is also exactly enough to buy 60 grapes. A recipe that makes 2 fruit salads calls for 2 apples and 14 grapes. How many fruit salads can I make?

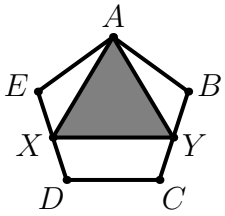
23. Alice and Bob are playing a guessing game. Alice picks a point in a two-dimensional space. Next, Bob guesses what he thinks this point is. Alice then tells Bob the distance between his point and the correct point. Assuming Bob guesses with a good strategy, what is the maximum number of guesses it could take for Bob to guess the correct point?

24. Sam, Jacob, and Kyle have the integers $1, 2, 3, \ldots, 30$ written on a whiteboard. They then erase some of the numbers as follows:

    - Sam erases all the multiples of 2 (i.e. $2, 4, 6, \ldots, 30$).
    - Of the remaining numbers, Jacob erases all the multiples of 3.
    - Of the remaining numbers, Kyle erases all the multiples of 5.

    How many integers remain on the whiteboard?

25. $ABCDE$ is a regular pentagon, as shown. Points X and Y are on sides BC and DE such that $\triangle AXY$ is equilateral, and line XY is parallel to line CD. What is the measure of $\angle AXE$ in degrees?



26. Five suspects, one of which committed a crime, are being investigated. They give the following statements:

    - Suspect 1: "It was either suspect 2 or 5."
    - Suspect 2: "It was neither me nor suspect 3."
    - Suspect 3: "Whatever suspect 4 says next is going to be incorrect."
    - Suspect 4: "Out of suspects 1 and 2, one of them told the truth and the other lied."
    - Suspect 5: "No, both suspects 1 and 2 lied."

    It is also known that exactly two of the suspects are lying. Which suspect committed the crime?

27. Let $n = 2022^{22}$. How many positive integers m are there such that m is a divisor of n, and n is a divisor of m^2?

28. Isaac, Albert, and Eli are attempting to guess Hari's secret number, which is a random integer from 1 to 12. Isaac guesses first; if he doesn't get it, then Albert guesses; if neither of them gets it, then Eli guesses; if the number

still has not been guessed, then it goes back to Isaac, and the cycle continues. Nobody ever guesses a number that has already been guessed. What is the probability that Isaac is the first to guess the secret number? Say that the probability is $\frac{a}{b}$ when written in lowest terms (i.e. when a and b are relatively prime). Submit the number $a + b$.

29. How many of the integers from 1 to 1000 are perfect squares but not perfect cubes?

30. Holden creates a sequence of numbers by starting with $a_1 = 2021$ and setting $a_n = 20 + a_{n-1}$ if n is even, and $a_n = 21a_{n-1}$ if n is odd. What are the last two digits of a_{2022}?

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